Towards complete string effective actions beyond leading order

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Abstract: We review the current knowledge of higher-derivative terms in string effective actions, the various approaches that have been used to obtain them and their applications.

1 Higher-derivative string effective actions

Many aspects of low-energy string dynamics can be captured in terms of a Wilsonian effective gravity field theory for the massless modes. Although it is often sufficient to consider only the lowest-order supergravity actions, there are qualitative string predictions for which knowledge about the subleading terms is required. Despite the fact that the first results about the structure of higher-derivative string effective actions are almost twenty years old, complete, supersymmetric invariants are still lacking, even at the first sub-leading order in α' beyond the supergravity level.

Four main techniques have been employed to derive supersymmetric higher-derivative string effective actions. One is to simply try to construct the most general supersymmetric invariant containing higher derivatives. Such an approach was pursued in the early days [1], but it is extremely cumbersome (even on a computer) due to the enormous number of terms and the problem of dealing with side relations such as Bianchi and Ricci identities, as well as partial integration. A second method employs the relation between string background field equations of motion and conformal invariance on the world-sheet. This method is very useful for the NS-NS sector of string theory, for which background field couplings are under control in the RNS formulation of the string, but it becomes much more complicated for R-R background fields. The latter can at present only be treated in the Berkovits formalism; the state of the art on such calculations can be found in [2]. Thirdly, the standard supergravity technique of solving superspace Bianchi identities is being pursued, with the higher-derivative terms arising from relaxed torsion constraints [3, 4]. While such an approach is potentially very useful for the construction of existence or uniqueness proofs concerning higher-derivative invariants, obtaining explicit component-space expressions in this way is very difficult. At present, the only method which has shown promise to be powerful enough to determine, in practise, the *entire* effective action in component form at sub-leading order is the construction directly from string scattering amplitudes. In the present letter we review the current status of this approach and comment on some related problems and often-raised questions.

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2 Killing spinors and supersymmetry-preserving solutions

One reason to study higher-derivative corrections to supergravity actions is that a knowledge of them would allow us to study corrections to supergravity solutions, e.g. black holes or D-branes. Such corrections are important because they influence entropy counting (for an analysis in four dimensions see [5]) or predictions of the string/gauge theory correspondence (see e.g. [6] for some initial steps in this direction). Many of these solutions contain non-trivial gauge-field configurations. A full analysis is at present seriously hampered by our incomplete understanding of the higher-derivative bosonic terms in the effective action.

Interesting solutions are those which preserve at least some fraction of supersymmetry, i.e. backgrounds in which the variations of the fermions vanish. For the gravitino in M-theory this takes, at classical level, the form

$$\delta_\epsilon \psi_\mu = \Big(\partial_\mu + \tfrac{1}{4}\omega_{\mu\nu_1\nu_2}\,\Gamma^{\nu_1\nu_2} + T_\mu{}^{\nu_1\cdots\nu_4}F_{\nu_1\ldots\nu_4}\Big)\epsilon = 0\,, \tag{1}$$

where $F = \mathrm{d}C$ and $T_\mu{}^{\nu_1\cdots\nu_4} = (\Gamma_\mu{}^{\nu_1\cdots\nu_4} - 8\delta_\mu{}^{\nu_1}\Gamma^{\nu_2\cdots\nu_4})/288$. This equation gives rise to a set of integrability conditions $[\mathcal{D}_\mu, \mathcal{D}_\nu]\epsilon = 0$. These equations, together with the equation of motion and Bianchi identity for the four-form, imply the equation of motion for the metric. The existence of a Killing spinor satisfying (1) imposes severe restrictions on the holonomy group associated with the vacuum solution, and restricts it to a subgroup of the Lorentz group.

Higher-derivative corrections to the action do, however, modify (1), because superinvariance at higher order means that there are corrections to the supersymmetry transformation rules: invariance of the action means that

$$\Big(\delta_0 + \sum_n (l_{\mathrm{P}})^n\delta_n\Big)\Big(S_0 + \sum_n (l_{\mathrm{P}})^n S_n\Big) = 0\,. \tag{2}$$

More explicitly, the Killing spinor equation receives corrections of the form

$$\delta\psi_\mu = \Big(\nabla_\mu + T_\mu{}^{\nu_1\cdots\nu_4}F_{\nu_1\ldots\nu_4}\Big)\epsilon + (l_{\mathrm{P}})^6\big(DR^3\epsilon\big)_\mu + (l_{\mathrm{P}})^6\big(\ldots\big)_\mu\,. \tag{3}$$

In certain situations, such as compactifications on large eight-manifolds, it is consistent to ignore all the l_{P} correction terms. This can be seen from a simple scaling argument. Under a scaling of the eight-manifold $g_{(8)} \to tg_{(8)}$ the first two terms scale as $t^{-3/2}$. The third term, which is the $C\wedge R^4$-induced correction to the supersymmetry transformation rules [7], instead scales as t^{-3}. Therefore it is perfectly consistent, for large t, to use only the lowest order Killing spinor equations (1), as was done in e.g. [8]. This is true despite the fact that the four-form equation of motion *does* receive a correction from the $C\wedge t_8 R^4$ term in the action,

$$\mathrm{d}{*}F_4 = \tfrac{1}{2}F_4\wedge F_4 + (l_{\mathrm{P}})^6\, t_8R^4 + (l_{\mathrm{P}})^6\big(\ldots\big)\,, \tag{4}$$

where the suppressed terms involve fields other than the graviton (and are at present completely unknown). Here the terms which are listed explicitly in (4) are all scale invariant. Any other terms, for instance those arising from an R^3F^2 term in the effective action, scale with some negative power of t and will therefore be suppressed in the large-volume limit. However, in general applications, these correction terms cannot be scaled away. Several corrections to the supersymmetry transformation rules have been computed

by us some time ago [7]. Incorporating the effects of higher-derivative terms and their influence on the holonomy structure group is still an open problem.

Given that the left-hand side of the equation of motion (4) is closed, it is natural to write it as

$$\mathrm{d}F_7 = \tfrac{1}{2}F_4 \wedge F_4 + (l_{\mathrm{P}})^6\, t_8 R^4 \tag{5}$$

(now meant to be read as an exact equation), supplemented by the duality relation

$$F_7 = *F_4 + (l_{\mathrm{P}})^6(\ldots)\,. \tag{6}$$

This is the point of view taken in [9], where the physically non-trivial deformations of this duality equation (or rather its superspace version) were analysed. In this form, an interesting parallel to the type-IIB theory arises [10]. Instead of having two gauge fields related by a duality condition, one there has a single gauge field with a self-duality condition. The analogue of the deformation of the duality relation between F_7 and F_4 now becomes a deformation of the self-duality condition of F_5 (or rather the composite field strength $\tilde{F}_5 = \mathrm{d}C_4 + \frac{1}{2}B_2 \wedge F_3 - \frac{1}{2}H_3 \wedge C_2$ with $H_3 = \mathrm{d}B_2$ and $F_3 = \mathrm{d}C_2$),

$$\tilde{F}_5 + (\alpha')^3 \frac{\delta S^{(3)}}{\delta \tilde{F}_5^+} = * \left[\tilde{F}_5 + (\alpha')^3 \frac{\delta S^{(3)}}{\delta \tilde{F}_5^+}\right]\,. \tag{7}$$

Some of these corrections have recently been computed by two of us [10].

3 Actions from string amplitudes

As stressed before, our approach is to derive effective actions directly from string theory amplitudes. At genus zero and one, the amplitudes with four external gravitons have been known for quite some time; these are relatively easy to compute and lead to the well-known fourth-order action

$$S = \int \mathrm{d}^{10}x\, \sqrt{-g} t_8 t_8 R^4\,. \tag{8}$$

This term is universal for all string theories. Beyond the four-point NS-NS sector, which also includes the two-form field, the situation quickly becomes much more complicated.

There are several reasons why such amplitudes are hard to compute. One of them is that the vertex operators for R-R gauge fields involve spin fields, the presence of which makes the worldsheet fermionic correlators difficult to evaluate. Fortunately, generic expressions have been derived [11] which completely resolve this problem and which avoid any explicit operator product expansions.

A second and more serious problem concerns the integration over the odd supermoduli, or more simply put, the integration over the inequivalent sectors of the world-sheet gravitino field. Consider a generic correlator, say at genus one in the odd/odd spin-structure sector, expanded in powers of the gauge-fixed world-sheet gravitinos χ_- and $\tilde{\chi}_+$:

$$\begin{aligned}\Big\langle V_1(z_1)\cdots V_n(z_n)\Big\rangle &= \int \mathcal{D}\chi\mathcal{D}\bar{\chi}\mathcal{D}X\mathcal{D}\Psi \,\Big[V_1(z_1)\cdots V_n(z_n)\Big]\, \exp\big(-S[X,\Psi]\big) \\ \times\, e^{\phi+\tilde{\phi}}\bigg(1 &- \frac{1}{2\pi\alpha'}\int \mathrm{d}^2z\, \tilde{\chi}_+\Psi\partial X(z) - \frac{1}{2\pi\alpha'}\int \mathrm{d}^2 z\, \chi_-\tilde{\Psi}\bar{\partial}X(z) + \frac{1}{4\pi\alpha'}\int \mathrm{d}^2z\, \tilde{\chi}_+\chi_-\Psi\tilde{\Psi}(z) \\ &+ \frac{1}{(2\pi\alpha')^2}\int \mathrm{d}^2 w \int \mathrm{d}^2 z\, \tilde{\chi}_+\Psi\partial X(w)\chi_-\tilde{\Psi}\bar{\partial}X(z)\bigg)\,.\end{aligned} \tag{9}$$

The Grassmann integrals select the terms with one χ_- and one $\tilde{\chi}_+$. One usually therefore only considers the last two terms in this expression, which have the structure of so-called "picture changing operators". However, this is only a correct procedure if there are no world-sheet gravitino modes in the vertex operators themselves, which is unfortunately *not* true [12] (see also the early work [13]). For e.g. the NS-NS two-form vertex operator, these additional terms are given by

$$V_B^{(0,0)} \quad = \quad V_B^{(0,0)}\Big|_{\text{standard}} \quad - \quad \frac{1}{6}\int \mathrm{d}^2 z\, H_{\mu\nu\rho}\Big(\Psi^\mu\Psi^\nu\Psi^\rho\tilde{\chi}_+ \quad - \quad \tilde{\Psi}^\mu\tilde{\Psi}^\nu\tilde{\Psi}^\rho\chi_-\Big)\, e^{ik_\rho X^\rho}\,. \tag{10}$$

Simple examples exist in which these gravitino terms really matter [12], and this issue becomes more complicated for increasing genus.

Once the correlators have been computed, one is still left with integrals over the modular parameters and vertex operator insertion points. Despite the loop-by-loop finiteness claim of string theory, these modular integrals are actually divergent for external momenta in the physical regime. This is a well-known problem, present already for the four-graviton amplitude at tree level [14] and typically circumvented by performing an analytic continuation in the Mandelstam variables, computing the integral in terms of standard functions, and then analytically continuing back. Unfortunately, this procedure in general requires that the integral is cut up in various pieces, which each have to be analytically continued in a different way [14]. This entire procedure makes it very hard to construct a systematic expansion in α' (let alone to do these integrals numerically). The origin of this problem lies in the Euclidean formulation of string perturbation theory [15].

Having resolved several of these problems in [12, 10], two of us have recently been able to perform the next step in the completion of the effective action (8) with other bosonic fields. To give a flavour of the form into which these results can be cast[1], we show here the form of the genus-one contribution to $W^2(DF^+_{(5)})^2$ terms in the effective action [10]:

$$\begin{aligned} S_{\rm IIB}^{W^2(DF^+_{(5)})^2} = \int \mathrm{d}^{10}x\,\sqrt{-g}\,\Big(&(16+\lambda)\,W^2\Big|_{\widetilde{[2,2,2,2]}} - 4(16-\lambda)\,W^2\Big|_{\widetilde{[2,2,2]}} + 192\,W^2\Big|_{\widetilde{[3,1,1]}} \\ &+ \tfrac{16}{15}(16+\lambda)\,W^2\Big|_{\widetilde{[2,2]}_A} + \tfrac{32}{3}\,W^2\Big|_{\widetilde{[4]}} + \tfrac{1}{21}(16+\lambda)\,W^2\Big|_{\widetilde{[2]}}\Big)\,(DF^+_{(5)}\Big|_{\widetilde{[2,1,1,1,1]}^+})^2\,. \end{aligned} \tag{11}$$

Here λ corresponds to a one-parameter ambiguity in mapping the string amplitudes to terms in the effective action.[2] This parameter is fixed by linearised supersymmetry to the value $\lambda = -16$, leaving only three non-vanishing contributions in (11).

4 Superspace and superspace constraints

As discussed at length in [7], there is a close connection between supersymmetry transformation rules and superspace torsion constraints: the algebra satisfied by the supersymmetry variations is isomorphic to that of the supercovariant derivatives, $\{D_a, D_b\} = T_{ab}{}^r D_r$.

[1] It should be mentioned that even when the relevant amplitudes have been calculated, the step of converting amplitudes to effective-action terms may be quite complicated. This is, in particular, true for the R-R bilinear higher-derivative terms to be discussed below, owing to the need to be able to recognise terms proportional to the lowest-order equations of motion.

[2] There are always ambiguities when translating on-shell string amplitudes to an effective action. Several terms with n powers of the fields may lead to vanishing on-shell n-point amplitudes, and in addition there is the freedom of field redefinition, which can be used to change the higher-order action by any term proportional to the lower-order field equations.

This link provides a way to obtain string-/M-theory corrections to the superspace geometries of ten- and eleven-dimensional supergravity theories through the construction of the relevant higher-derivative supersymmetry invariants. Such corrections are, for instance, of central importance for the study of higher-derivative corrections to kappa-symmetric world-volume actions of D-branes and M-branes.

A crucial step in this programme to obtain corrections to the torsion constraints from the component formalism was made in [7]. This involved the use of string input to cast the $d = 10$, $N = 1$ supersymmetric completion (originally derived in [1]) of the term

$$I_X = t_8 t_8 R^4 + \tfrac{1}{2}\varepsilon_{10} B t_8 R^4 \tag{12}$$

in a compact "t_8" form, together with the associated modifications of the supersymmetry transformations of the basic fields. These were subsequently lifted to eleven dimensions. It was found, however, that incorporating the I_X superinvariant does not lead to any modifications of the superspace geometry. Using cohomological methods in superspace [16], it has subsequently been shown [9] that any modification of eleven-dimensional supergravity that does not involve a non-vanishing lowest-dimensional component of the four-form superfield, is trivial. Hence, in a component-space approach, we expect that the inclusion (and supersymmetrisation) of higher-derivative gauge-field terms is necessary in order to get explicit results for the corrections to the superspace geometry. It is also still possible that such corrections arise from the supersymmetrisation of the term

$$I_Z = -\varepsilon_{10}\varepsilon_{10} R^4 + 4\varepsilon_{10} B t_8 R^4\,, \tag{13}$$

appearing together with I_X in the type-II effective actions. In [12] some partial results in this direction were presented. More specifically, it was shown how a careful treatment of left/right-mixing zero-mode terms in the bosonic two-point functions on the torus is necessary to resolve an otherwise puzzling issue regarding the cancellation mechanism for supersymmetry variations of the anomaly term.

Finally, let us comment briefly on the chiral superfield of the type-IIB theory [17], which has been used to construct a superinvariant at the linearised level. If this construction is extended to the non-linear level a chiral measure is required [18] in order to perform the integration over the odd coordinates. Such a chiral measure is known *not* to exist as there exists only one chiral superfield [17]. However, this does not necessarily prevent the construction of an on-shell non-linear superinvariant, as the modified torsion constraints relate the variations of the measure (or rather the full higher-derivative part of the action) to the lowest-order supergravity action. While examples do exist in which a linearised supergravity action based on chiral superfields requires the introduction of non-chiral fields at the non-linear level (e.g. $N = 3$ conformal supergravity), all of these involve only lowest order actions and none involve mixing through modified torsion constraints. In any case, the scalar superfield is somewhat of a curiosity in the larger scheme of things, and the only rigorous way to obtain information about non-linear terms in the effective action at the component level is, at present, by computing them directly from string theory.

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